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Extremal $(n, n+1)$-graphs with respected to zeroth- order general Randić index

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A $(n, n+1)$-graph G is a connected simple graph with n vertices and $n+1$ edges. If d_v denotes the degree of the vertex v, then the zeroth-order general Randić index $R_\alpha^0(G)$ of the graph G is defined as $\sum_{v\in V(G)} d_v^\alpha$, where α is a real number. We characterize, for any α, the $(n, n+1)$-graphs with the smallest and greatest zeroth-order general Randić index.

KEY WORDS: (n, n+1)-graph, zeroth-order general Randic index, degree sequence

1. Introduction

Let $G = (V, E)$ be a simple connected graph with the vertex set V and the edge set E. For any $v \in V$, $N(v)$ denotes the neighbors of v, and $d_v = |N(v)|$ is the degree of v. The Randić index (or connectivity index) of G was introduced by Randić in 1975 and defined as [1]

$$R(G) = \sum_{uv\in E} (d_u d_v)^{-1/2}.$$

Randić demonstrated that his index is well correlated with a variety of physic-chemical properties of various classes of organic compounds. Eventually, the index $R(G)$ has become one of the most popular molecular descriptors (see [2–4]).

In [5], generalized $R(G)$ by replacing the exponent $-1/2$ by an arbitrary real number α. This graph invariant is called the general Randić index and denoted by R_α, i.e., $R_\alpha(G) = \sum_{uv\in E} (d_u d_v)^\alpha$. Li and Yang [6] studied R_α for all graphs of order n and characterized the corresponding extremal graphs. Later, Hu et al. [7, 8] determined the trees with extremal R_α, and Li et al. [9] gave the

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lower and upper bounds for R_α of chemical (n, m)-graphs. Other results of the general Randić index can be found in [10].

The zeroth-order Randić index defined by Kier and Hall [11] is

$$R^0(G) = \sum_{v \in V(G)} d_v^{-1/2}.$$

Pavlović [12] determined the unique graph with maximum value of $R^0(G)$. Lang et al. [13] investigated the same problem for the topological index $M_1(G)$, also known as Zagreb index, which is defined as $M_1(G) = \sum_{v \in V(G)} d_v^2$. Eventually, Li and Zheng [14] defined the zeroth-order general Randić index of a graph G as

$$R_\alpha^0(G) = \sum_{v \in V(G)} d_v^\alpha$$

for any real number α. Li and Zhao [10] characterized trees with the first three largest and smallest zeroth-order general Randić indices and Wang and Deng [15] characterized the unicycle graphs with the maximum zeroth-order general Randić index with the exponent α being equal to m, $-m$, $1/m$, $-1/m$, where $m \geqslant 2$ is an integer. Hua and Deng [16] characterized the unicycle graphs with the maximum and minimum zeroth-order general Randić index for any real number α.

In this paper, we investigate the zeroth-order general randić index $R_\alpha^0(G)$ of $(n, n+1)$-graphs G, i.e., connected simple graphs with n vertices and $n+1$ edges. We characterize the $(n, n+1)$-graphs with extremal (maximum or minimum) zeroth-order general Randić index.

Note that if $\alpha = 0$ then $R_\alpha^0(G) = n$, and if $\alpha = 1$ then $R_\alpha^0(G) = 2m$. Therefore, in the following we always assume that $\alpha \neq 0, 1$.

First, we need to introduce some transformations.

Denote by $D(G) = [d_1, d_2, \ldots, d_n]$ the degree sequence of the graph G, where d_i stands the degree of the ith vertex of G, and $d_1 \geqslant d_2 \geqslant \cdots \geqslant d_n$.

If there is a graph G, such that $d_i \geqslant d_j + 2$. Let G' be the graph obtained from G by replacing the pair (d_i, d_j) by the pair $(d_i - 1, d_j + 1)$. In other words, if $D(G) = [d_1, d_2, \ldots, d_{i-1}, d_i, d_{i+1}, \ldots, d_{j-1}, d_j, d_{j+1}, \ldots, d_n]$, then $D(G') = [d_1, d_2, \ldots, d_{i-1}, d_i - 1, d_{i+1}, \ldots, d_{j-1}, d_j + 1, d_{j+1}, \ldots, d_n]$.

Lemma 1.1 ([17]). For the two graphs G and G', specified above, we have

(i) $R_\alpha^0(G) > R_\alpha^0(G')$ for $\alpha < 0$ or $\alpha > 1$;

(ii) $R_\alpha^0(G) < R_\alpha^0(G')$ for $0 < \alpha < 1$.

From lemma 1.1, we have immediately

Theorem 1.1. Let G_0 be a $(n, n+1)$-graph with degree sequence $[d_1, d_2, \ldots, d_n]$, such that $|d_i - d_j| \leqslant 1$ for any $i \neq j$. Then for $\alpha < 0$ or $\alpha > 1$, G_0 has the minimum zeroth-order general Randić index among all $(n, n+1)$-graphs, whereas for $0 < \alpha < 1$, G_0 has the maximum zeroth-order general Randić index among all $(n, n+1)$-graphs.

In fact, G_0 is one of the graphs in figure 1.

Transformation A: If there are two vertices u and v in G such that $d_u = p > 1$, $d_v = q > 1$, and $p \leqslant q$. $u_1, u_2, \ldots, u_k$ are adjacent to u, $G' = G - \{uu_1, uu_2, \ldots, uu_k\} + \{vu_1, vu_2, \ldots, vu_k\}$, $1 \leqslant k \leqslant p$, as shown in figure 2.

Lemma 1.2. For the two graphs G and G' above, we have

(i) $R^0_\alpha(G') > R^0_\alpha(G)$ for $\alpha < 0$ or $\alpha > 1$;

(ii) $R^0_\alpha(G') < R^0_\alpha(G)$ for $0 < \alpha < 1$.

Proof. By the definition of $R^0_\alpha(G)$, we have

$$\begin{aligned} \Delta &= R^0_\alpha(G') - R^0_\alpha(G) \\ &= [(p-k)^\alpha + (q+k)^\alpha] - [p^\alpha + q^\alpha] \\ &= [(q+k)^\alpha - q^\alpha] - [p^\alpha - (p-k)^\alpha] \\ &= \alpha \cdot k(\xi^{\alpha-1} - \eta^{\alpha-1}), \end{aligned}$$

where $\eta \in (p-k, p)$, $\xi \in (q, q+k)$. $\xi > \eta$ since $p \leqslant q$. Then $\Delta > 0$ when $\alpha > 1$ or $\alpha < 0$; $\Delta < 0$ when $0 < \alpha < 1$. So, the proof of lemma 1.2 is completed.

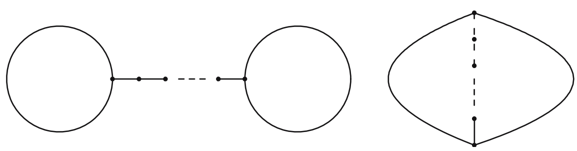

Figure 1.

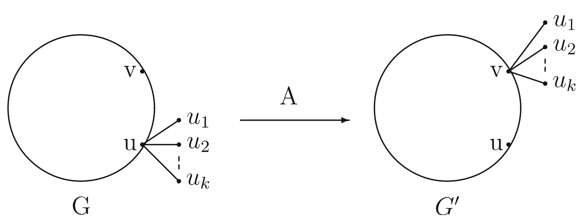

Figure 2.

Transformation B: Let uv be an edge G, the degree $d_G(u)$ of u is $p > 1$, $N_G(v)$ is the neighborhood of v and $N_G(v) - \{u\} = \{w_1, w_2, \ldots, w_l\}$. $G^{'} = G - \{vw_1, vw_2, \ldots, vw_l\} + \{uw_1, uw_2, \ldots, uw_l\}$, as shown in figure 3.

Lemma 1.3. For the two graphs G and G' above, we have

(i) $R_\alpha^0(G^{'}) > R_\alpha^0(G)$, for $\alpha > 1$ or $\alpha < 0$;

(ii) $R_\alpha^0(G^{'}) < R_\alpha^0(G)$, for $0 < \alpha < 1$.

Proof. If $p \geqslant l + 1$, then

$$\begin{aligned}\Delta &= R_\alpha^0(G^{'}) - R_\alpha^0(G)\\ &= [(p+l)^\alpha + 1] - [p^\alpha + (l+1)^\alpha]\\ &= [(p+l)^\alpha - p^\alpha] - [(l+1)^\alpha - 1^\alpha]\\ &= \alpha l(\xi^{\alpha-1} - \eta^{\alpha-1}) \qquad (\eta \in (1, l+1), \xi \in (p, p+l)).\end{aligned}$$

If $p \leqslant l + 1$, then

$$\begin{aligned}\Delta &= R_\alpha^0(G^{'}) - R_\alpha^0(G)\\ &= [(p+l)^\alpha + 1] - [p^\alpha + (l+1)^\alpha]\\ &= [(p+l)^\alpha - (l+1)^\alpha] - [p^\alpha - 1^\alpha]\\ &= \alpha(p-1)(\xi^{\alpha-1} - \eta^{\alpha-1}) \qquad (\eta \in (1, p), \xi \in (l+1, p+l)).\end{aligned}$$

So, $\xi > \eta$ and $\Delta > 0$ when $\alpha > 1$ or $\alpha < 0$; $\Delta < 0$ when $0 < \alpha < 1$. The proof of lemma 1.3 is completed.

Note that the zeroth-order general Randić index will increase for $\alpha > 1$ or $\alpha < 0$ and decrease for $0 < \alpha < 1$ by using transformations A or B.

Now, we need to introduce some notations.

Let $\mathcal{G}(n, n+1)$ be the set of simple connected graphs with n vertices and $n + 1$ edges.

For any graph $G \in \mathcal{G}(n, n+1)$, there are two cycles C_p and C_q in G.

(1) $\mathcal{A}(p, q)$ is the set of $G \in \mathcal{G}(n, n+1)$ in which the cycles C_p and C_q have only one common vertex;

(2) $\mathcal{B}(p, q)$ is the set of $G \in \mathcal{G}(n, n+1)$ in which the cycles C_p and C_q have no common vertex;

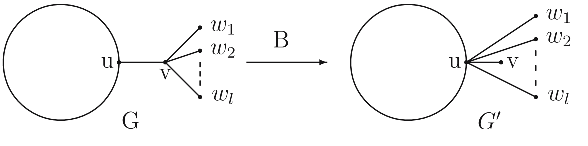

Figure 3.

(3) $\mathcal{C}(p,q,l)$ is the set of $G \in \mathcal{G}(n,n+1)$ in which the cycles C_p and C_q have a common path of length l.

Note that the induced subgraph of vertices on the cycles of $G \in \mathcal{A}(p,q)$ (or $\mathcal{B}(p,q)$, $\mathcal{C}(p,q,l)$) is showed in figure 4(a) (or (b),(c)) and $\mathcal{C}(p,q,l) = \mathcal{C}(p, p+q-2l, p-l) = \mathcal{C}(p+q-2l, q, q-l)$.

2. Extremal graphs in $\mathcal{A}(p,q)$

Let $S_n(p,q)$ be the graph in $\mathcal{A}(p,q)$ such that $n+1-p-q$ pendent edges attach to the common vertex of C_p and C_q.

Lemma 2.1. For any graph $G \in \mathcal{A}(p,q)$, if G is not isomorphic to $S_n(p,q)$, then

(i) $R_\alpha^0(S_n(p,q)) > R_\alpha^0(G)$, for $\alpha > 1$ or $\alpha < 0$;

(ii) $R_\alpha^0(S_n(p,q)) < R_\alpha^0(G)$, for $0 < \alpha < 1$.

Proof. Repeating Transformation B, any graph $G \in \mathcal{A}(p,q)$ can be changed into a graph G' in which the edges not in the cycles C_p and C_q are pendent edges, by lemma 1.3, $R_\alpha^0(G') > R_\alpha^0(G)$ (for $\alpha > 1$ or $\alpha < 0$) or $R_\alpha^0(G') < R_\alpha^0(G)$ (for $0 < \alpha < 1$). And repeating transformation A, G' can be changed into a graph G'' in which these pendent edges are attached to the same vertex u, by lemma 1.2, $R_\alpha^0(G'') > R_\alpha^0(G')$ (for $\alpha > 1$ or $\alpha < 0$) or $R_\alpha^0(G'') < R_\alpha^0(G')$ (for $0 < \alpha < 1$). If u is not the common vertex of C_p and C_q, i.e., $G'' \neq S_n(p,q)$, then $R_\alpha^0(S_n(p,q)) > R_\alpha^0(G'')$ (for $\alpha > 1$ or $\alpha < 0$) or $R_\alpha^0(S_n(p,q)) < R_\alpha^0(G'')$ (for $0 < \alpha < 1$) from

$$\begin{aligned} &R_\alpha^0(S_n(p,q)) - R_\alpha^0(G'') \\ &= [(k+4)^\alpha + 2^\alpha] - [(k+2)^\alpha + 4^\alpha] \\ &= [(k+4)^\alpha - (k+2)^\alpha] - [4^\alpha - 2^\alpha] \\ (or\ &= [(k+4)^\alpha - 4^\alpha] - [(k+2)^\alpha - 2^\alpha]), \end{aligned}$$

where $k = n+1-p-q$.

Lemma 2.2 (1). If $p > 3$, then

(i) $R_\alpha^0(S_n(p,q)) > R_\alpha^0(S_n(p-1,q))$, for $\alpha > 1$ or $\alpha < 0$;

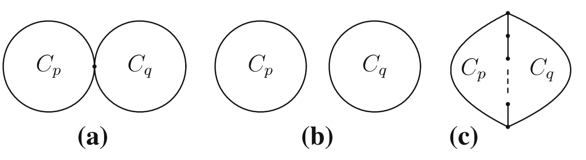

Figure 4.

(ii) $R_\alpha^0(S_n(p,q)) < R_\alpha^0(S_n(p-1,q))$, for $0 < \alpha < 1$;

(2) If $q > 3$, then

(i) $R_\alpha^0(S_n(p,q)) > R_\alpha^0(S_n(p,q-1))$, for $\alpha > 1$ or $\alpha < 0$;

(ii) $R_\alpha^0(S_n(p,q)) < R_\alpha^0(S_n(p,q-1))$, for $0 < \alpha < 1$.

Proof. (1) By the definition of $R_\alpha^0(G)$, we have

$$\begin{aligned}\Delta &= R_\alpha^0(S_n(p-1,q)) - R_\alpha^0(S_n(p,q))\\ &= [(n+6-p-q)^\alpha + 1^\alpha] - [(n+5-p-q)^\alpha - 2^\alpha]\\ &= [(n+6-p-q)^\alpha - (n+5-p-q)^\alpha] - [2^\alpha - 1^\alpha]\\ &= \alpha(\xi^{\alpha-1} - \eta^{\alpha-1}) \qquad (\xi \in (n+5-p-q, n+6-p-q); \eta \in (1,2)).\end{aligned}$$

Then $\Delta > 0$ when $\alpha > 1$ or $\alpha < 0$; $\Delta < 0$ when $0 < \alpha < 1$. So, the proof of lemma 2.2 is completed.

(2) It can be proved like (1).

By lemmas 2.1 and 2.2, we have

Theorem 2.1. If $\alpha > 1$ or $\alpha < 0$, then the graph in $\mathcal{A}(p,q)$ with maximum zeroth-order general Randić index is $S_n(p,q)$, and the graph in $\mathcal{A}(p,q)$ for all $p \geqslant 3$ and $q \geqslant 3$ with maximum zeroth-order general Randić index is $S_n(3,3)$.

If $0 < \alpha < 1$, then the graph in $\mathcal{A}(p,q)$ with minimum zeroth-order general Randić index is $S_n(p,q)$, and the graph in $\mathcal{A}(p,q)$ for all $p \geqslant 3$ and $q \geqslant 3$ with minimum zeroth-order general Randić index is $S_n(3,3)$.

3. Extremal graphs in $\mathcal{B}(p,q)$

Lemma 3.1. For any graph $G \in \mathcal{B}(p,q)$, if the length of the shortest path connecting C_p and C_q in G is r, then

(i) $R_\alpha^0(T_n^r(p,q)) > R_\alpha^0(G)$, for $\alpha > 1$ or $\alpha < 0$;

(ii) $R_\alpha^0(T_n^r(p,q)) < R_\alpha^0(G)$, for $0 < \alpha < 1$,

where $T_n^r(p,q)$ is obtaining from connecting C_p and C_q by a path P of length r and the other edges are all attached to the same end-vertex of P (see figure 5 (a) or (b)).

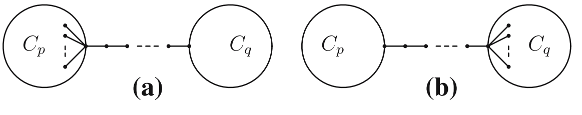

Figure 5.

Proof. Repeating transformation B, the graph $G \in B(p,q)$ can be changed into a graph G' in which the edges not in the cycles C_p, C_q and P are pendent edges, by lemma 1.3, $R_\alpha^0(G') > R_\alpha^0(G)$ (for $\alpha > 1$ or $\alpha < 0$) or $R_\alpha^0(G') < R_\alpha^0(G)$ (for $0 < \alpha < 1$). And repeating transformation A, G' can be changed into a graph G'' in which these pendent edges are attached to the same vertex u, by lemma 1.2, $R_\alpha^0(G'') > R_\alpha^0(G')$ (for $\alpha > 1$ or $\alpha < 0$) or $R_\alpha^0(G'') < R_\alpha^0(G')$ (for $0 < \alpha < 1$). If u is not a end-vertex of P, then $R_\alpha^0(T_n^r(p,q)) > R_\alpha^0(G'')$ (for $\alpha > 1$ or $\alpha < 0$) or $R_\alpha^0(T_n^r(p,q)) < R_\alpha^0(G'')$ (for $0 < \alpha < 1$) from

$$\begin{aligned}&R_\alpha^0(T_n^r(p,q)) - R_\alpha^0(G'')\\&= [(k+3)^\alpha + 2^\alpha] - [(k+2)^\alpha + 3^\alpha]\\&= [(k+3)^\alpha - (k+2)^\alpha] - [3^\alpha - 2^\alpha],\end{aligned}$$

where $k = n + 1 - p - q - r$.

Lemma 3.2. (1) If $p > 3$, then

(i) $R_\alpha^0(T_n^r(p-1,q)) > R_\alpha^0(T_n^r(p,q))$, for $\alpha > 1$ or $\alpha < 0$;

(ii) $R_\alpha^0(T_n^r(p-1,q)) < R_\alpha^0(T_n^r(p,q))$, for $0 < \alpha < 1$;

(2) If $q > 3$, then

(i) $R_\alpha^0(T_n^r(p,q-1)) > R_\alpha^0(T_n^r(p,q))$, for $\alpha > 1$ or $\alpha < 0$;

(ii) $R_\alpha^0(T_n^r(p,q-1)) < R_\alpha^0(T_n^r(p,q))$, for $0 < \alpha < 1$;

(3) If $r > 1$, then

(i) $R_\alpha^0(T_n^{r-1}(p,q)) > R_\alpha^0(T_n^r(p,q))$, for $\alpha > 1$ or $\alpha < 0$;

(ii) $R_\alpha^0(T_n^{r-1}(p,q)) < R_\alpha^0(T_n^r(p,q))$, for $0 < \alpha < 1$.

Proof. (1) By the definition of $R_\alpha^0(G)$, we have

$$\begin{aligned}\Delta &= R_\alpha^0(T_n^r(p-1,q)) - R_\alpha^0(T_n^r(p,q))\\&= [(n+5-p-q)^\alpha + 1^\alpha] - [(n+4-p-q)^\alpha - 2^\alpha]\\&= [(n+5-p-q)^\alpha - (n+4-p-q)^\alpha] - [2^\alpha - 1^\alpha]\\&= \alpha(\xi^{\alpha-1} - \eta^{\alpha-1}) \qquad (\xi \in (n+4-p-q, n+5-p-q); \eta \in (1,2)).\end{aligned}$$

Then $\Delta > 0$ when $\alpha > 1$ or $\alpha < 0$; $\Delta < 0$ when $0 < \alpha < 1$. So, the proof of lemma 3.2 is completed.

(2) and (3) can be proved like (1).

By lemmas 3.1 and 3.2, we have

Theorem 3.1. If $\alpha > 1$ or $\alpha < 0$, then the graph in $\mathcal{B}(p,q)$ with maximum zeroth-order general Randić index is $T_n^1(p,q)$, and the graph in $\mathcal{B}(p,q)$ for all $p \geqslant 3$ and $q \geqslant 3$ with maximum zeroth-order general Randić index is $T_n^1(3,3)$.

If $0 < \alpha < 1$, then the graph in $\mathcal{B}(p, q)$ with minimum zeroth-order general Randić index is $T_n^1(p, q)$, and the graph in $\mathcal{B}(p, q)$ for all $p \geqslant 3$ and $q \geqslant 3$ with minimum zeroth-order general Randić index is $T_n^1(3, 3)$.

4. Extremal graphs in $\mathcal{C}(p, q, l)$

As in sections 3, we have the following results and omit their proofs here.

Lemma 4.1. For any graph $G \in \mathcal{C}(p, q, l)$, we have

(i) $R_\alpha^0(\theta_n^l(p, q)) > R_\alpha^0(G)$, for $\alpha > 1$ or $\alpha < 0$;

(ii) $R_\alpha^0(\theta_n^l(p, q)) < R_\alpha^0(G)$, for $0 < \alpha < 1$

where $\theta_n^l(p, q)$ is obtaining from the graph in figure 4(c) by attaching $n + l + 1 - p - q$ edges to one of its vertices with degree 3.

Lemma 4.2. (1) If $p > 3$, then

(i) $R_\alpha^0(\theta_n^l(p - 1, q)) > R_\alpha^0(\theta_n^l(p, q))$, for $\alpha > 1$ or $\alpha < 0$;

(ii) $R_\alpha^0(\theta_n^l(p - 1, q)) < R_\alpha^0(\theta_n^l(p, q))$, for $0 < \alpha < 1$;

(2) If $q > 3$, then

(i) $R_\alpha^0(\theta_n^l(p, q - 1)) > R_\alpha^0(\theta_n^l(p, q))$, for $\alpha > 1$ or $\alpha < 0$;

(ii) $R_\alpha^0(\theta_n^l(p, q - 1)) < R_\alpha^0(\theta_n^l(p, q))$, for $0 < \alpha < 1$;

(3) If $l > 1$, then

(i) $R_\alpha^0(\theta_n^{l-1}(p, q)) > R_\alpha^0(\theta_n^l(p, q))$, for $\alpha > 1$ or $\alpha < 0$;

(ii) $R_\alpha^0(\theta_n^{l-1}(p, q)) < R_\alpha^0(\theta_n^l(p, q))$, for $0 < \alpha < 1$.

Theorem 4.2. If $\alpha > 1$ or $\alpha < 0$, then the graph in $\mathcal{C}(p, q, l)$ with maximum zeroth-order general Randić index is $\theta_n^l(p, q)$, and the graph in $\mathcal{C}(p, q, l)$ for all $p \geqslant 3$, $q \geqslant 3$ and $l > 1$ with maximum zeroth-order general Randić index is $\theta_n^1(3, 3)$.

If $0 < \alpha < 1$, then the graph in $\mathcal{C}(p, q, l)$ with minimum zeroth-order general Randić index is $\theta_n^l(p, q)$, and the graph in $\mathcal{C}(p, q, l)$ for all $p \geqslant 3$, $q \geqslant 3$ and $l > 1$ with minimum zeroth-order general Randić index is $\theta_n^1(3, 3)$.

5. Extremal graphs in $\mathcal{G}(n, n + 1)$

Theorem 5.1. (1). $\theta_n^1(3, 3)$ is the graph with maximum zeroth-order general Randić index in $\mathcal{G}(n, n + 1)$ for $\alpha > 1$ or $\alpha < 0$;

(2) $\theta_n^1(3,3)$ is also the graph with minimum zeroth-order general Randić index in $\mathcal{G}(n, n+1)$ for $0 < \alpha < 1$.

Proof. By theorems in sections 2–4, we only need compare the zeroth-order general Randić indices of $S_n(3,3)$, $T_n^1(3,3)$, and $\theta_n(3,3)$.

$$\begin{aligned}\Delta_1 &= R_\alpha^0(\theta_n^1(3,3)) - R_\alpha^0(S_n(3,3))\\ &= [(n-1)^\alpha + 3^\alpha + 2\times 2^\alpha + (n-4)] - [(n-l)^\alpha + 4\times 2^\alpha + (n-5)]\\ &= [3^\alpha - 2^\alpha] - [2^\alpha - 1^\alpha]\\ &= \alpha(\xi^{\alpha-1} - \eta^{\alpha-1}) \qquad (\xi \in (2,3), \eta \in (1,2)),\end{aligned}$$

$$\begin{aligned}\Delta_2 &= R_\alpha^0(\theta_n^1(3,3)) - R_\alpha^0(T_n^1(3,3))\\ &= [(n-1)^\alpha + 3^\alpha + 2\times 2^\alpha + (n-4)] - [(n-3)^\alpha + 4\times 2^\alpha + 3^\alpha + (n-6)]\\ &= [(n-1)^\alpha - (n-3)^\alpha] - 2[2^\alpha - 1^\alpha]\\ &= 2\alpha(\xi^{\alpha-1} - \eta^{\alpha-1}) \qquad (\xi \in (n-3, n-1), \eta \in (1,2)),\end{aligned}$$

$$\begin{aligned}\Delta_3 &= R_\alpha^0(S_n(3,3)) - R_\alpha^0(T_n^1(3,3))\\ &= [(n-1)^\alpha + 4\times 2^\alpha + (n-5)] - [(n-3)^\alpha + 4\times 2^\alpha + 3^\alpha + (n-6)]\\ &= [(n-1)^\alpha - (n-3)^\alpha] - [3^\alpha - 1^\alpha]\\ &= 2\alpha(\xi^{\alpha-1} - \eta^{\alpha-1}) \qquad (\xi \in (n-3, n-1), \eta \in (1,3)).\end{aligned}$$

Then $\Delta_i > 0$ when $\alpha > 1$ or $\alpha < 0$; $\Delta_i < 0$ when $0 < \alpha < 1$ $(i = 1,2,3)$. So, the proof of theorem 5.1 is completed.

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